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FLOER HOMOLOGY OF BRIESKORN HOMOLOGY SPHERES

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Abstract

Every Brieskorn homology sphere $\Sigma(p,q,r)$ is a double cover of the 3–sphere ramified over a Montesinos knot $k(p,q,r)$. We express the Floer homology of $\Sigma(p,q,r)$ in terms of certain invariants of the knot $k(p,q,r)$, among which are the knot signature and the Jones polynomial. We also define an integer valued invariant of integral homology 3–spheres which agrees with the $\bar{\mu}$–invariant of W. Neumann and L. Siebenmann for Seifert fibered homology spheres, and investigate its behavior with respect to homology 4–cobordism.

Let p, q, and r be pairwise coprime positive integers. A Brieskorn homology 3–sphere $\Sigma(p,q,r)$ is the link of the singularity of $f^{-1}(0)$, where $f : \mathbb{C}^3 \to \mathbb{C}$ is a map of the form $f(x,y,z) = x^p + y^q + z^r$. The complex conjugation in $\mathbb{C}^3$ acts on $\Sigma(p,q,r)$ turning it into a double branched cover of S^3 branched over a Montesinos knot $k(p,q,r)$.

The Floer homology groups $I_n(\Sigma)$, $0 \le n \le 7$, are abelian groups associated with an integral homology 3–sphere Σ; see [14]. The Floer homology of $\Sigma(p,q,r)$ was studied by R. Fintushel and R. Stern [13] who showed in particular that the groups $I_*(\Sigma(p,q,r))$ are free abelian, and if a_n denotes the rank of $I_n(\Sigma(p,q,r))$ then $a_n = 0$ for odd n. Therefore,

$$a_0 + a_2 + a_4 + a_6 = 2\,\lambda(\Sigma(p,q,r)), \tag{1}$$

where $\lambda(\Sigma(p,q,r))$ is the Casson invariant; see [36]. We add to this knowledge the following result:

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