ARITHMETIC MANIFOLDS OF POSITIVE SCALAR CURVATURE

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1. Introduction

Gromov and Lawson [18] and Schoen and Yau [34] have shown that no compact manifold of nonpositive sectional curvature can be endowed with a metric of positive scalar curvature. As is very well recognized by now, their approach is actually based on a restriction on the coarse quasi-isometry type of complete noncompact manifolds of positive scalar curvature. Our goal in this paper is to explore the situation if we study the problem of complete metrics with no quasiisometry conditions at all.

Let M be an irreducible locally symmetric space of noncompact type of finite volume. It is the double coset space $\Gamma \backslash G/K$ associated to a lattice Γ in a semisimple Lie group G. Our main theorem is the following:

Theorem 1.1. *Let $M = \Gamma \backslash G/K$, G semisimple and Γ an irreducible lattice. M can be given a complete metric of uniformly positive scalar curvature $\kappa \geq \epsilon > 0$ if and only if Γ is an arithmetic group of $\mathbb{Q}$-rank at least 3.*

Amplification 1.2.