

BOUNDARY SLOPES OF IMMERSSED SURFACES IN 3-MANIFOLDS

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Abstract

This paper presents some finiteness results for the number of boundary slopes of immersed proper π_1 -injective surfaces of given genus g in a compact 3-manifold with torus boundary. In the case of hyperbolic 3-manifolds we obtain uniform quadratic bounds in g , independent of the 3-manifold.

1. Introduction

An immersed, proper, π_1 -injective surface in a compact 3-manifold M with non-empty boundary is *essential* if it cannot be properly homotoped into ∂M . Let c be a homotopically non-trivial simple loop in ∂M . If there is a proper immersion of an essential surface F into M such that each component of ∂F is homotopic to a multiple of c , we call c a *boundary slope* of M . The first question we look at is a problem of P. Shalen, told to us by M. Baker:

Question 1. Does the set of essential surfaces with bounded genus in a simple knot complement give rise to at most finitely many boundary slopes?

Baker has given examples to show that if the bounded genus assumption is dropped, then infinitely many boundary slopes can be realized [5], and Oertel has found examples of manifolds in which every slope is realized by the boundary of an immersed essential surface [18], see also [25]. On the other hand, Hatcher [13] has shown that there are only finitely many boundary slopes for embedded essential surfaces, without a genus restriction.

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