

ASSIGNMENTS AND ABSTRACT MOMENT MAPS

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Abstract

Abstract moment maps arise as a generalization of genuine moment maps on symplectic manifolds when the symplectic structure is discarded, but the relation between the mapping and the action is kept. Particular examples of abstract moment maps had been used in Hamiltonian mechanics for some time, but the abstract notion originated in the study of cobordisms of Hamiltonian group actions.

In this paper we answer the question of existence of a (proper) abstract moment map for a torus action and give a necessary and sufficient condition for an abstract moment map to be associated with a pre-symplectic form. This is done by using the notion of an assignment, which is a combinatorial counterpart of an abstract moment map.

Finally, we show that the space of assignments fits as the zeroth cohomology in a series of certain cohomology spaces associated with a torus action on a manifold. We study the resulting “assignment cohomology” theory.

1. Introduction

Abstract moment maps arise as a generalization of genuine moment maps on symplectic manifolds. The essence of their definition is that the symplectic structure is discarded, but the relation between the mapping and the action is kept intact. To be precise, an abstract moment map on a G -manifold M is an equivariant mapping $\Psi: M \rightarrow \mathfrak{g}^*$ satisfying the following constancy condition: for every Lie algebra element $\xi \in \mathfrak{g}$, the component $\langle \Psi, \xi \rangle$ is locally constant on the set $\{\xi_M = 0\}$ where the action generating vector field ξ_M vanishes.

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