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TWISTING AND NONNEGATIVE CURVATURE METRICS ON VECTOR BUNDLES OVER THE ROUND SPHERE

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Abstract

A complete noncompact manifold M with nonnegative sectional curvature is diffeomorphic to the normal bundle of a compact submanifold S called the soul of M. When S is a round sphere we show that the clutching map of this bundle is restricted; this is used to deduce that there are at most finitely many isomorphism types of such bundles with sectional curvature lying in a fixed interval $[0, \kappa]$. We also examine the opposite question of how the twisting of the bundle limits the type of possible nonnegative curvature metrics on the bundle: It turns out that if the bundle does not admit a nowhere-zero section, then the normal exponential map is necessarily a diffeomorphism onto M, and the ideal boundary of M consists of a single point.

In their paper [3], Cheeger and Gromoll raised the question of which vector bundles over the round sphere admit complete metrics with nonnegative sectional curvature. The significance of this problem is that it attempts to determine to what extent a converse to the Soul theorem holds. Recall that this theorem states that every open (i.e., complete noncompact) manifold M with nonnegative curvature K_M is diffeomorphic to a vector bundle over a compact totally geodesic submanifold S called a soul. A natural question then is whether all such vector bundles admit complete metrics with $K_M \geq 0$.

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