J. DIFFERENTIAL GEOMETRY
52 (1999) 173-187

VIRTUALLY HAKEN DEHN-FILLING

D. COOPER & D. D. LONG

Abstract

We show that "most" Dehn-fillings of a non-fibered, atoroidal, Haken three-manifold with torus boundary are virtually Haken.

1. Results

Suppose that X is a compact, oriented, three-manifold with boundary a torus T. We will pick a basis of $H_1(T)$ represented by simple loops λ, μ such that $\lambda = 0$ in $H_1(X; \mathbf{Q})$. We call λ a *longitude* and μ a *meridian.* A *slope*, α, on T is the isotopy class of an essential unoriented simple closed curve. The manifold $X(\alpha)$ is the result of Dehn-filling along the slope α. This means that a solid torus is glued along its boundary to T so that a meridian disc of the solid torus is glued onto α. The manifold X is *atoroidal* if every $\mathbf{Z} \times \mathbf{Z}$ subgroup of $\pi_1 X$ is conjugate into $\pi_1 T$. The *distance* between two slopes α, β is $\Delta(\alpha, \beta)$ which is the absolute value of the algebraic intersection number of the homology classes represented by these slopes.

Theorem 1.1. *Suppose that X is a compact, connected, oriented, irreducible, atoroidal three-manifold with boundary a torus T. Suppose that S is a compact, connected, oriented, non-separating, incompressible surface properly embedded in X with non-empty boundary. Suppose that S is not a fiber of a fibration of X over the circle. Let g be the genus*

Received July 7, 1997. Both authors were partially supported by the NSF.
Keywords: 3-manifold, Dehn-filling, virtually Haken.
AMS subject classification: 57M25, 57M50.