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HÖLDER REGULARITY OF HOROCYCLE FOLIATIONS

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1. Introduction

Let M be a C^∞, nonpositively curved manifold. A *horosphere* in M is the projection to M of a limit of metric spheres in the universal cover $\tilde{M}$ (see §2). A *horospherical foliation* $\mathcal{H}$ is a foliation of the unit tangent bundle T^1M whose leaves consist of unit normal vector fields to horospheres.[1]

While regularity of horospherical foliations has been studied extensively for negatively curved manifolds M, considerably less is known in the nonpositively curved case. The most general result is due to P. Eberlein: if M is complete and nonpositively curved, then horospheres are C^2, which implies that the individual leaves of $\mathcal{H}$ are C^1. Further, the tangent distribution $T\mathcal{H}$ is continuous on T^1M (see [9]).

Beyond Eberlein's theorem, smoothness results have consisted mainly of counterexamples ([2], [5]); in particular, the best one could hope for in the case of a general compact, nonpositively curved M is for $T\mathcal{H}$ to be Hölder-continuous. In this paper we prove

Theorem I$'$. *Let S be a compact, real-analytic, nonpositively curved surface. Then $T\mathcal{H}$ is Hölder.*

Theorem I$'$ is actually a corollary of a more general result, Theorem I below.

The problem of finding the regularity of horospherical foliations has a long history, which we briefly summarize here.

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[1]As we explain in §2, there are two such foliations, $\mathcal{H}^-$ and $\mathcal{H}^+$, called *stable* and *unstable* horospherical foliations, respectively. In this discussion, we use $\mathcal{H}$ to denote either of these.