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AN ANALYTIC COMPACTIFICATION OF SYMPLECTIC GROUP

HONGYU L. HE

Introduction

In this paper, we propose an analytic compactification from the symplectic group $Sp_{2n}(\mathbb{R})$ to the symmetric space $U(2n)/O_{2n}(\mathbb{R})$. We obtain this compactification through the study of Bargmann-Segal model of the oscillator (metaplectic) representation.

In the theory of symmetric spaces, a Hermitian symmetric space of noncompact type can be realized as a bounded domain in a complex vector space. For example, $SL(2,\mathbb{R})/SO(2)$ can be realized as the Poincaré disc. Harish-Chandra studied harmonic analysis on such a domain. The studies along this line had been quite fruitful for holomorphic discrete representations. For a noncompact reductive group G, we also wish to do analysis on an appropriate compactification $\overline{G}$ of G. If the "push forward" of matrix coefficients of unitary representations of G behaves reasonably well, we may gain a better understanding of unitary representations of G through the study of functions on $\overline{G}$ (see [5]).

To begin with, let X be an analytic manifold. We say $(i, \overline{X})$ is an analytic compactification of X, if $\overline{X}$ is a compact analytic manifold and

$$i : X \to \overline{X}$$

is an analytic embedding such that $i(X)$ is open dense in $\overline{X}$. Let G be the standard symplectic group. Then G has a KAK decomposition, where K is $U(n) = Sp_{2n}(\mathbb{R}) \cap SO_{2n}(\mathbb{R})$ and $A \cong \mathbb{R}^n$. Let K^o be the opposite group. Then G has a $K \times K^o$ action. For the symmetric

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