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Gr $\Longrightarrow$ SW
FROM PSEUDO-HOLOMORPHIC CURVES TO SEIBERG–WITTEN SOLUTIONS

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The Seiberg-Witten invariants were defined by Witten [24] for any compact, oriented 4-manifold with $b^2_+ > 1$; after the choice of an orientation for a certain determinant line, they consitute an map, SW, from the set $\mathcal{S}$ of Spin$^{\mathbb{C}}$ structures on X to $\mathbb{Z}$ which depends only on the diffeomorphism type of X. Roughly speaking, SW is computed from a weighted count of solutions to a natural, non-linear system of differential equations on X. (See [9], [8] and [12].) As remarked in [17], a symplectic 4-manifold has a canonical identification $\mathcal{S} \approx H^2(X;\mathbb{Z})$; and with this identification understood, the Seiberg-Witten invariant on a symplectic X can be thought of as mapping $H^2(X;\mathbb{Z})$ to $\mathbb{Z}$.

Meanwhile, a symplectic 4-manifold has a second natural map, Gr: $H^2(X;\mathbb{Z}) \to \mathbb{Z}$, called the Gromov invariant. The latter invariant is defined in [18]. To a first approximation, Gr assigns to a class $e \in H^2(X;\mathbb{Z})$ a certain weighted count of the symplectic submanifolds of X whose fundamental class is Poincaré dual to e. The following theorem was announced in [17]:

Theorem 1. *Let X be a compact, symplectic 4-manifold with $b^2_+ > 1$. Use the symplectic structure to orient X, to define the Seiberg-Witten invariants of X as a map $SW : H^2(X;\mathbb{Z}) \to \mathbb{Z}$, and also to define the Gromov invariant $Gr : H^2(X;\mathbb{Z}) \to \mathbb{Z}$. Then $Gr = SW$.*

As remarked in [17], there are essentially three parts to the proof of this theorem. The first part appears in [19] where it was shown how

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