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THE NIETO QUINTIC IS JANUS-LIKE

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Introduction

The Nieto quintic, which we denote by $\mathcal{N}_5$ in this paper, is a very special quintic hypersurface in $\mathbb{P}^4 = \{\sum_{i=0}^5 x_i = 0\} \subset \mathbb{P}^5$ given by the equations

$$\mathcal{N}_5 := \{\sum_{i=0}^{5} x_i = \sum_{i \neq j \neq k \neq l \neq m} x_i x_j x_k x_l x_m = 0\} \subset \mathbb{P}^5.$$

It is the most singular member of the pencil of quintics which are invariant under the natural action of the symmetric group Σ_6 acting by permutation of coordinates. This quintic hypersurface was discovered by Nieto in his thesis [10], as a closely related variety to the variety which parametrizes those lines in $\mathbb{P}^3$ which lie on some smooth Heisenberg-invariant quartic surface. The Nieto quintic is the Hessian variety of the Segre cubic (cf. [8, 3.2]) $\mathcal{S}_3$, which in its turn is the unique cubic invariant under Σ_6. The singular locus of $\mathcal{N}_5$ consists of the following: 10 isolated nodes p_i; 20 singular lines ℓ_k which meet in 15 singular points q_j. Each of the points q_j has multiplicity 3.

In [8, 3.4.4] I made a conjecture concerning a certain birational image of $\mathcal{N}_5$, to the effect that the image is (precisely) the Satake compactification of a ball quotient, and it is that conjecture which will be verified in this paper. We identify modular subvarieties and compactification locus, and that is what leads to the above: the compactification divisors of the ball quotient will consist of the proper transforms of the ten points p_i as well as the proper transforms of the 20 singular lines ℓ_k.

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