J. DIFFERENTIAL GEOMETRY
50 (1998) 505-518

SYMPLECTIC RATIONAL BLOWDOWNS

MARGARET SYMINGTON

Abstract

We prove that the rational blowdown, a surgery on smooth 4-manifolds introduced by Fintushel and Stern, can be performed in the symplectic category. As a consequence, interesting families of smooth 4-manifolds, including the exotic $K3$ surfaces of Gompf and Mrowka, admit symplectic structures.

A basic problem in symplectic topology is to understand what smooth manifolds admit a symplectic structure (a closed non-degenerate 2-form). In this paper we focus on this question in dimension 4. Currently, the primary methods for constructing smooth (irreducible) 4-manifolds in such a way that one can distinguish them by Donaldson or Seiberg-Witten invariants are surgery constructions which use complex manifolds as building blocks. These surgery methods are (smooth) logarithmic transforms, rational blowdowns, and connect sums along surfaces. It is interesting to see when these surgeries can be performed in the symplectic category. In this paper we prove that performing a rational blowdown of a symplectic manifold along symplectic surfaces yields a symplectic manifold. This result establishes that certain exotic 4-manifolds, including the exotic $K3$ surfaces of Gompf and Mrowka [9], are symplectic.

In any even dimension, two symplectic manifolds can be summed along codimension 2 symplectic submanifolds to yield a new symplectic manifold. We refer to this symplectic operation, which was proposed by Gromov [11], as the symplectic sum. Gompf [8] used the symplectic sum to construct a plethora of interesting symplectic manifolds, including the first examples of simply connected symplectic 4-manifolds that are

Received April 16, 1998. The author is grateful for the support of an NSF postdoctoral fellowship, DSM9627749.