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# HAMILTONIAN LOOP GROUP ACTIONS AND VERLINDE FACTORIZATION

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## Abstract

We prove a formula for $\mathrm{Spin}_c$ quantizations of reductions of Hamiltonian actions of loop groups. This includes as a special case the factorization formula for the $\mathrm{Spin}_c$ quantization of the moduli space of flat connections over a compact oriented two-manifold.

## 1. Introduction

The geometric quantization of the moduli space of flat connections over a surface has been the subject of intensive study from a number of different points of view. Much of the recent work in mathematics has focused on proving formulas discovered by the physicist E. Verlinde [51] in the context of conformal field theory. The two ingredients in Verlinde's approach are the "factorization property", which describes the behavior of the quantization when boundary circles are glued together, and the "fusion rules", which describe the quantization of the moduli space of a three-holed sphere (pair of pants) with boundary components marked by irreducible representations of the loop group.

In the Kähler approach to geometric quantization, one takes the quantization to be the space of sections of a pre-quantum line bundle that are holomorphic with respect to some Kähler structure. In this setting both parts of Verlinde's approach were carried out rigorously by Tsuchiya-Ueno-Yamada [48], using degeneration of a chosen conformal structure on the surface. For further information on this approach,

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