

ON THE STRUCTURE OF SPACES WITH RICCI CURVATURE BOUNDED BELOW. II

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0. Introduction

This paper, the sequel of [4], is the second in a series devoted to the study of the structure of complete connected riemannian manifolds, M^n , whose Ricci curvature has a definite lower bound and of the Gromov-Hausdorff limits, Y , of sequences of such manifolds.

By [4], in the noncollapsed case, off a subset of codimension ≥ 2 , such a limit space, Y , is bi-Hölder equivalent to a connected smooth riemannian manifold (for the proof of connectedness, see Section 3 below). Additionally, even in the collapsed case, there exist natural renormalized limit measures, ν , with respect to which Y is infinitesimally Euclidean almost everywhere. We do not know whether “bi-Hölder” can be replaced by “bi-Lipschitz”, or “infinitesimally Euclidean” by “locally Euclidean”. Nor do we know whether in the collapsed case, the local Hausdorff dimension of the space is the same at all points.

In order to describe the results of the present paper in detail, we will recall some background from [4].

After rescaling the metric, we can assume

$$(0.1) \quad \text{Ric}_{M^n} \geq -(n-1).$$

Sometimes we assume in addition that for some definite $v > 0$,

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