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CONNECTIONS IN POISSON GEOMETRY I: HOLONOMY AND INVARIANTS

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Abstract

We discuss contravariant connections on Poisson manifolds. For vector bundles, the corresponding operational notion of a contravariant derivative had been introduced by I. Vaisman. We show that these connections play an important role in the study of global properties of Poisson manifolds and we use them to define Poisson holonomy and new invariants of Poisson manifolds.

Introduction

Let M be a Poisson manifold and suppose that we require the existence of a linear connection on M, compatible with the Poisson tensor Π. Since parallel transport preserves the rank of the Poisson tensor, the Poisson manifold must be regular in order for such a connection to exist. Therefore, the usual notion of a covariant connection is not appropriate for the study of Poisson manifolds, as some of the most interesting examples of Poisson manifolds are non-regular. For non-regular Poisson manifolds the symplectic foliation is singular and the dimension of the leaves varies, so one can only hope to compare tangent spaces at different points of the same symplectic leaf.

One possible way around this difficulty is to use families of connections parameterized by the leaves. However, there are examples showing that the symplectic foliation can be wild, so the space of leaves will not be easy to parameterize.

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