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ON THE HOMOTOPY INVARIANCE OF HIGHER SIGNATURES FOR MANIFOLDS WITH BOUNDARY

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Abstract

If M is a compact oriented manifold-with-boundary whose fundamental group is virtually nilpotent or Gromov-hyperbolic, we show that the higher signatures of M are oriented-homotopy invariants. We give applications to the question of when higher signatures of closed manifolds are cut-and-paste invariant.

0. Introduction

The Novikov Conjecture hypothesizes that certain numerical invariants of closed oriented manifolds, called higher signatures, are oriented-homotopy invariants. It is natural to ask if there is an extension of the Novikov Conjecture to manifolds with boundary. Such an extension was made in [27], [30]. In this paper we show that if the relevant discrete group is virtually nilpotent or Gromov-hyperbolic then the higher signatures defined in [27], [30] are oriented-homotopy invariants.

Before giving our result, let us recall the statement of Novikov's conjecture. Let M be a closed oriented smooth manifold. Let $L \in \mathrm{H}^*(M;\mathbb{Q})$ be the Hirzebruch L-class and let $*L \in \mathrm{H}_*(M;\mathbb{Q})$ be its Poincaré dual. If Γ is a finitely-generated discrete group, let $B\Gamma$ denote its classifying space. Recall that $\mathrm{H}^*(B\Gamma;\mathbb{Q}) \cong \mathrm{H}^*(\Gamma;\mathbb{Q})$, the rational group cohomology of Γ. Let $\nu : M \to B\Gamma$ be a continuous map, defined up to

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