

NONNEGATIVE RICCI CURVATURE, SMALL LINEAR DIAMETER GROWTH AND FINITE GENERATION OF FUNDAMENTAL GROUPS

CHRISTINA SORMANI

1. Introduction

In 1968, Milnor conjectured that a complete noncompact manifold, M^n , with nonnegative Ricci curvature has a finitely generated fundamental group [11]. This was proven for a manifold with nonnegative sectional curvature by Cheeger and Gromoll [6]. However, it remains an open problem even for manifolds with strictly positive Ricci curvature.

The conjecture is of particular interest because, if it is true, then by work of Cheeger-Gromoll, Milnor and Gromov, the fundamental group is almost nilpotent [7], [11], [8]. On the other hand, given any finitely generated torsion free nilpotent group, Wei has constructed an example of a manifold with positive Ricci curvature that has the given group as a fundamental group [15].

Schoen and Yau have proven the conjecture in dimension 3 for manifolds with strictly positive Ricci curvature [13]. In fact they have proven that such a manifold is diffeomorphic to Euclidean space.

Anderson and Li have each proven that if a manifold with nonnegative Ricci curvature has Euclidean volume growth, then the fundamental group is actually finite [2], [10]. Anderson uses volume comparison arguments while Li uses the heat equation to prove this theorem.

Abresch and Gromoll have proven that manifolds with small diameter growth, $(o(r^{1/n}))$, nonnegative Ricci curvature and sectional curvature bounded away from negative infinity have finite topological type [1, Thm. A]. Thus the fundamental group is finitely generated in such

Received October 21 1999.