J. DIFFERENTIAL GEOMETRY
79 (2008) 335-387

RIGIDITY OF AMALGAMATED PRODUCTS IN NEGATIVE CURVATURE

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Abstract

Let Γ be the fundamental group of a compact riemannian manifold X of sectional curvature $K \leq -1$ and dimension $n \geq 3$. We suppose that $\Gamma = A *_C B$ is the free product of its subgroups A and B amalgamated over the subgroup C. We prove that the critical exponent $\delta(C)$ of C satisfies $\delta(C) \geq n-2$. The equality occurs if and only if there exist an embedded compact hypersurface $Y \subset X$, totally geodesic, of constant sectional curvature -1, whose fundamental group is C and which separates X in two connected components whose fundamental groups are A and B respectively. Similar results hold if Γ is an HNN extension, or more generally if Γ acts on a simplicial tree without fixed point.

1. Introduction

In [**17**], Y. Shalom proved the following theorem that says that for every lattice Γ in the isometry group of the hyperbolic space and for any decomposition of Γ as an amalgamated product $\Gamma = A *_C B$, the group C has to be "big". In order to measure how "big" C is, let us define the critical exponent of a discrete group C acting on a Cartan Hadamard manifold by

$$\delta(C) = \inf\{s > 0 \quad | \quad \Sigma_{\gamma\in\Gamma} e^{-sd(\gamma x, x)} < +\infty\}.$$

Theorem 1.1 ([**17**]). *Let Γ be a lattice in $PO(n,1)$. Assume that Γ is an amalgamated product of its subgroups A and B over C. Then, the critical exponent $\delta(C)$ of C satisfies $\delta(C) \geq n-2$.*

An example is given by any n-dimensional hyperbolic manifold X which contains a compact separating connected totally geodesic hypersurface Y. The Van Kampen theorem then says that the fundamental group Γ of X is isomorphic to the free product of the fundamental groups of the two halves of $X - Y$ amalgamated over the fundamental group C of the incompressible hypersurface Y. Such examples do exist; this is the case in dimension 3 thanks to the W.Thurston's hyperbolization theorem. In these cases there is equality in Theorem 1.1, i.e., $\delta(C) = n-2$

Received 09/22/2005.