J. DIFFERENTIAL GEOMETRY
79 (2008) 243-275

ON THE GENUS OF TRIPLY PERIODIC MINIMAL SURFACES

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Abstract

We prove the existence of embedded minimal surfaces of arbitrary genus $g \geq 3$ in any flat 3-torus. In fact, we construct a sequence of such surfaces converging to a planar foliation of the 3-torus. In particular, the area of the surface can be chosen arbitrarily large.

1. Introduction

Triply periodic minimal surfaces in euclidean space are invariant by three independant translations. In the nineteenth century, five embedded triply periodic minimal surfaces were known to H.A. Schwarz and his school. In 1970, Alan Schoen described 12 further families of examples. The arguments were completed by H. Karcher, who also proved the existence of many further examples.

If M is a triply periodic minimal surface and Λ is the lattice generated by its three periods, then M projects to a minimal surface in the flat 3-torus $\mathbb{R}^3/\Lambda$. Conversely, a (non-flat) minimal surface in $\mathbb{R}^3/\Lambda$ lifts to a triply periodic minimal surface in $\mathbb{R}^3$.

A natural question is whether there exist non-flat minimal surfaces in any flat 3-torus. The examples constructed by H.A. Schwarz, A. Schoen and H. Karcher are very symmetric by construction, so they only construct examples in very particular 3-tori. In 1990, W. Meeks made the following conjecture ([**6**], Conjecture 3.2):

Conjecture 1 (Meeks)**.** For any flat 3-torus $\mathbb{R}^3/\Lambda$ and any integer $g \geq 3$ there exist an embedded, orientable minimal surface of genus g in $\mathbb{R}^3/\Lambda$.

A related question asked by H. Karcher is the following ([**4**], question 4):

Can triply periodic embedded minimal surfaces of arbitrary large genus exist in a nontrivial way – or does the genus stay bounded if one divides out by all translational symmetries?

Received 12/21/2006.