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ON THE STEINNESS OF A CLASS OF KÄHLER MANIFOLDS

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Abstract

Let (M^n, g) be a complete non-compact Kähler manifold with non-negative and bounded holomorphic bisectional curvature. We prove that M is holomorphically covered by a pseudoconvex domain in $\mathbb{C}^n$ which is homeomorphic to $\mathbb{R}^{2n}$, provided (M^n, g) has uniform linear average quadratic curvature decay.

1. Introduction

Let (M^n, g_0) be a complete non-compact Kähler manifold with complex dimension n and with bounded nonnegative holomorphic bisectional curvature. Let R be the scalar curvature, and define

$$k(x,r) := \frac{1}{V_x(r)} \int_{B_x(r)} R dV.$$

In [**8**], it was proved by the authors that if M has maximum volume growth, then M is biholomorphic to $\mathbb{C}^n$. There, the authors used a result of Ni in [**22**] (see also [**10**, **13**]), which states that the condition of maximum volume growth on M implies that

$$k(x,r) \leq \frac{C}{1+r^2} \tag{1.1}$$

for some C for all x and r. In [**9**], the authors proved that condition (1.1) implies that M is holomorphically covered by $\mathbb{C}^n$, without assuming the maximum volume growth condition. The proof is obtained by studying the Kähler-Ricci flow

$$\frac{dg_{i\bar{j}}}{dt} = -R_{i\bar{j}} \tag{1.2}$$

with initial data g_0. It is well-known by [**30**] that if the scalar curvature decays linearly in the average sense:

$$k(x,r) \leq C/(1+r) \tag{1.3}$$

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