J. DIFFERENTIAL GEOMETRY
79 (2008) 141-165

MAXIMUM PRINCIPLES AT INFINITY

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Abstract

We prove a general maximum principle at infinity for properly immersed minimal surfaces with boundary in $\mathbb{R}^3$. An important corollary of this maximum principle at infinity is the existence of a fixed sized regular neighborhood for any properly embedded minimal surface of bounded curvature.

1. Introduction

Maximum principles play a fundamental and unifying role in the development of many deep results in geometry and analysis. Special cases of these general maximum principles, applied at infinity, have been used in an essential way in classifying genus zero minimal surfaces in $\mathbb{R}^3$ and in $\mathbb{R}^3/\Lambda$, where Λ is a discrete rank 2 subgroup (see [**12**, **15**, **17**]). The main goal of this paper is to develop general maximum principles at infinity for embedded complete minimal and constant mean curvature surfaces that include all previous results of this type and that are sufficiently powerful to have important applications to the global theory of these surfaces. Results of this type and the barrier type arguments used here first appeared in the proof of the strong halfspace theorem in [**9**] and in the isometric classification of properly embedded minimal surfaces in $\mathbb{R}^3$ (see [**4**]). A maximum principle at infinity for minimal surfaces of finite total curvature first appeared in [**11**]. A significant generalization was done in [**16**]. The explicit statement of the Maximum Principle at Infinity appears in Theorem 5.1.

Some clever barrier arguments are also used to prove maximum principles at infinity to study problems in other areas of differential geometry. Some earlier fundamental works on this topic can be found in the papers of Ni and Tam [**19**], Omari [**20**], and Yau [**25**]

Our paper is organized as follows. In Section 2, we prove that an injective minimal immersion of a complete surface of bounded Gaussian curvature into $\mathbb{R}^3$ is a proper mapping. In Section 3, we prove some

This material is based upon work for the NSF under Award No. DMS - 0405836. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the NSF.

Received 01/09/2006.