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SPECTRUM OF THE LAPLACIAN ON QUATERNIONIC KÄHLER MANIFOLDS

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Abstract

Let M^{4n} be a complete quaternionic Kähler manifold with scalar curvature bounded below by $-16n(n+2)$. We get a sharp estimate for the first eigenvalue $\lambda_1(M)$ of the Laplacian, which is $\lambda_1(M) \le (2n+1)^2$. If the equality holds, then either M has only one end, or M is diffeomorphic to $\mathbb{R} \times N$ with N given by a compact manifold. Moreover, if M is of bounded curvature, M is covered by the quaterionic hyperbolic space $\mathbb{QH}^n$ and N is a compact quotient of the generalized Heisenberg group. When $\lambda_1(M) \ge \frac{8(n+2)}{3}$, we also prove that M must have only one end with infinite volume.

0. Introduction

Let M^n be a complete n-dimensional Riemannian manifold with Ricci curvature bounded below by $-(n-1)$. It is well known from Cheng [**Ch**] that the first eigenvalue $\lambda_1(M)$ satisfies

$$\lambda_1(M) \le \frac{(n-1)^2}{4}.$$

In [**LW3**], Li and Wang proved an analogous theorem for complete Kähler manifolds. They showed that if M^{2n} is a complete Kähler manifold of complex dimension n with holomorphic bisectional curvature BK_M bounded below by -1, then the first eigenvalue $\lambda_1(M)$ satisfies

$$\lambda_1(M) \le n^2.$$

Here $\mathrm{BK}_M \ge -1$ means that

$$R_{i\bar{i}j\bar{j}} \ge -(1+\delta_{ij})$$

for any unitary frame $e_1, \ldots, e_n$.

In this paper, we prove the corresponding Laplacian comparison theorem for a quaterionic Kähler manifold M^{4n}. As an application we get

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