J. DIFFERENTIAL GEOMETRY
78 (2008) 13-32

CALABI-YAU CONNECTIONS WITH TORSION ON TORIC BUNDLES

D. Grantcharov, G. Grantcharov, & Y.S. Poon

Abstract

We find sufficient conditions for principal toric bundles over compact Kähler manifolds to admit Calabi-Yau connections with torsion, as well as conditions to admit strong Kähler connections with torsion. With the aid of a topological classification, we construct such geometry on $(k-1)(S^2\times S^4)\#k(S^3\times S^3)$ for all $k\geq 1$.

1. Introduction

In this article, we investigate a construction of Hermitian connections with special holonomy on Hermitian non-Kählerian manifolds. On Hermitian manifolds, there is a one-parameter family of Hermitian connections canonically depending on the complex structure J and the Riemannian metric g [**22**]. Among them is the Chern connection on holomorphic tangent bundles. In this paper, we are interested in what physicists call the Kähler-with-torsion connection (a.k.a. KT connection) [**41**]. It is the unique Hermitian connection whose torsion tensor is totally skew-symmetric when 1-forms are identified to their dual vectors with respect to the Riemannian metric. If T is the torsion tensor of a KT connection, it is characterized by the identity [**22**]

$$g(T(A,B),C)=dF(JA,JB,JC)$$

where F is the Kähler form; $F(A,B)=g(JA,B)$, and A,B,C are any smooth vector fields.

As a Hermitian connection, the holonomy of a KT connection is contained in the unitary group $\mathrm{U}(n)$. If the holonomy of the KT connection is reduced to $\mathrm{SU}(n)$, the Hermitian structure is said to be Calabi-Yau with torsion (a.k.a. CYT).

Such geometry in physical context was considered first by A. Strominger [**41**] and C. Hull [**32**]. More recently CYT structures on non-Kähler manifolds attracted attention as models for string compactifications. Many examples were found [**5**], [**2**], [**14**], [**13**], [**24**], [**26**]. This led

The second author was partially supported by NSF DMS-0333172. The third author was partially supported by NSF DMS-0204002.
Received 07/25/2005.