J. DIFFERENTIAL GEOMETRY
77 (2007) 135-148

KOHN-ROSSI COHOMOLOGY AND ITS APPLICATION TO THE COMPLEX PLATEAU PROBLEM, II

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Dedicated to Professor J.J. Kohn on the occasion of his 75th birthday

Abstract

Let X be a compact connected strongly pseudoconvex CR Manifold of real dimension $2n-1$ in $\mathbb{C}^{n+1}$. Tanaka introduced a spectral sequence $E_r^{(p,q)}(X)$ with $E_1^{(p,q)}(X)$ being the Kohn-Rossi cohomology group and $E_2^{(k,0)}(X)$ being the holomorphic De Rham cohomology denoted by $H_h^k(X)$. We study the holomorphic De Rham cohomology in terms of the s-invariant of the isolated singularities of the variety V bounded by X. We give a characterization of the singularities with vanishing s-invariants. For $n \geq 3$, Yau used the Kohn-Rossi cohomology groups to solve the classical complex Plateau problem in 1981. For $n = 2$, the problem has remained unsolved for over a quarter of a century. In this paper, we make progress in this direction by putting some conditions on X so that V will have very mild singularities. Specifically, we prove that if $\dim X = 3$ and $H_h^2(X) = 0$, then X is a boundary of complex variety V with only isolated quasi-homogeneous singularities such that the dual graphs of the exceptional sets in the resolution are star shaped and all curves are rational.

1. Introduction

One of the natural fundamental questions of complex geometry is to study the boundaries of complex varieties. For example, the famous classical complex Plateau problem asks which odd dimensional real submanifolds of $\mathbb{C}^N$ are boundaries of complex submanifolds in $\mathbb{C}^N$. In 1975, Harvey and Lawson [**Ha-La**] showed that for any compact connected CR manifold X in $\mathbb{C}^N$, there is a unique complex variety V in $\mathbb{C}^N$ such that the boundary of V is X. Therefore a natural and important question is to study V in terms of X explicitly.

The research was partially supported by RGC Hong Kong. The second author was partially supported by NSF.

Received 07/20/2005.