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CONCENTRATED, NEARLY MONOTONIC, EPIPERIMETRIC MEASURES IN EUCLIDEAN SPACE

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Abstract

We characterize Hölder continuously differentiable m dimensional submanifolds of Euclidean space among m rectifiable sets S in terms of growth conditions on the m density ratios of the Hausdorff measure $\mathcal{H}^m \llcorner S$.

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1. Foreword

We consider an m dimensional differentiable submanifold $S \subset \mathbf{R}^n$, $0 < m < n$, whose tangent spaces $\operatorname{Tan}(S,x)$ vary Hölder continuously with respect to $x \in S$. In other words there are $0 < \alpha \leq 1$ and $C_1 > 0$ such that

$$\operatorname{dist}(\operatorname{Tan}(S,x_1), \operatorname{Tan}(S,x_2)) \leq C_1 |x_1 - x_2|^\alpha$$

whenever $x_1, x_2 \in S$. Here $\operatorname{dist}(W_1, W_2)$ measures the distance between two m dimensional vector subspaces of $\mathbf{R}^n$; for instance we may set it equal to the Hilbert-Schmidt norm of $P_{W_1} - P_{W_2}$ where P_W denotes the nearest point projection on W. Our purpose is to study the *measure of area* on S, that is the Radon measure $\phi = \mathcal{H}^m \llcorner S$ defined as follows:

$$(\mathcal{H}^m \llcorner S)(A) = \mathcal{H}^m(S \cap A)$$

whenever $A \subset \mathbf{R}^n$. We have denoted by $\mathcal{H}^m$ the m dimensional Hausdorff measure on S, see e.g., [**10**, 2.10.2], so that ϕ "coincides with the Lebesgue measure in coordinate charts" according to the area theorem, [**10**, 3.2.3]. We observe (Proposition 3.6.1) that each $x_0 \in S$ has a neighborhood U with the following property (here $\operatorname{spt}(\phi) = S$).

(A) For every $x \in \operatorname{spt}(\phi) \cap U$ and every $0 < r < R$ such that $\mathbf{B}(x,R) \subset U$ one has:

$$\left| \frac{\phi(\mathbf{B}(x,r))}{\boldsymbol{\alpha}(m) r^m} - \frac{\phi(\mathbf{B}(x,R))}{\boldsymbol{\alpha}(m) R^m} \right| \leq C_2 R^{2\alpha}.$$

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