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GLOBAL RIGIDITY OF 3-DIMENSIONAL CONE-MANIFOLDS

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Abstract

We prove global rigidity for compact hyperbolic and spherical cone-3-manifolds with cone-angles $\leq \pi$ (which are not Seifert fibered in the spherical case), furthermore for a class of hyperbolic cone-3-manifolds of finite volume with cone-angles $\leq \pi$, possibly with boundary consisting of totally geodesic hyperbolic turnovers. To that end we first generalize the local rigidity result contained in [**Wei**] to the setting of hyperbolic cone-3-manifolds of finite volume as above. We then use the techniques developed in [**BLP**] to deform the cone-manifold structure to a complete non-singular or a geometric orbifold structure, where global rigidity holds due to Mostow-Prasad rigidity, cf. [**Mos**], [**Pra**], in the hyperbolic case, resp. [**deR**], cf. also [**Rot**], in the spherical case. This strategy has already been implemented successfully by [**Koj**] in the compact hyperbolic case if the singular locus is a link using Hodgson-Kerckhoff local rigidity, cf. [**HK**].

1. Introduction

Let X be a compact, orientable hyperbolic (resp. spherical) cone-3-manifold with cone-angles $\leq \pi$. Let $\Sigma = \cup_{i=1}^{N} e_i$ be the singular locus and $M = X \setminus \Sigma$ the smooth part of X, let further $(\alpha_1, \ldots, \alpha_N)$ be the vector of cone-angles. Due to the above assumptions on the cone-angles, a component of Σ will either be a (connected) trivalent graph or a circle embedded geodesically into X. M carries a smooth Riemannian metric of sectional curvature -1 in the hyperbolic case, resp. $+1$ in the spherical case, which is necessarily incomplete (the metric completion of M being given by glueing the singular locus back in).

Recall that cone-3-manifolds of curvature $\kappa \in \mathbb{R}$ with cone-angles $\leq 2\pi$ are complete metric length spaces with curvature bounded from below by κ in the triangle comparison sense. The local structure of a cone-3-manifold X of curvature κ is fixed by prescribing local models: For any point $x \in X$ the metric ball of sufficiently small radius centered at x is required to be isometric to a truncated cone of curvature κ over a spherical cone-surface, which will be called the (spherical) link of x.

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