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ON THE CLASSIFICATION OF LORENTZIAN HOLONOMY GROUPS

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Abstract

If an $(n+2)$-dimensional Lorentzian manifold is indecomposable, but non-irreducible, then its holonomy algebra is contained in the parabolic algebra $(\mathbb{R}\oplus\mathfrak{so}(n))\ltimes\mathbb{R}^n$. We show that its projection onto $\mathfrak{so}(n)$ is the holonomy algebra of a Riemannian manifold. This leads to a classification of Lorentzian holonomy groups and implies that the holonomy group of an indecomposable Lorentzian spin manifold with parallel spinor equals to $G \ltimes \mathbb{R}^n$ where G is a product of $SU(p)$, $Sp(q)$, G_2 or $Spin(7)$.

1. Introduction

Holonomy groups. An important tool to study the geometric structure of a smooth manifold M equipped with a linear connection ∇ is its *holonomy group*. Parallel sections in geometric vector bundles, such as tensor products of the tangent bundle or the spin bundle, correspond to invariant objects under the holonomy representation. By a result of J. Hano and H. Ozeki [**22**] any closed subgroup of $Gl(m,\mathbb{R})$ can be obtained as a holonomy group of a connection, but possibly a connection with torsion. By imposing conditions on the torsion there arises a classification problem of possible holonomy groups. In order to tackle such a classification problem one usually assumes that the connection is torsion free and that holonomy group acts irreducibly. If the connection is torsion free, its curvature satisfies the Bianchi-identity imposing algebraic constraints to the holonomy algebra via the Ambrose-Singer holonomy theorem [**3**]. By evaluating these constraints M. Berger classified the irreducible semi-Riemannian holonomy groups (see [**6**] for not locally symmetric semi-Riemannian manifolds, [**7**] for symmetric spaces, and [**35**], [**2**], [**10**] and [**11**] for simplifications, corrections and existence results in the Riemannian case), while L. Schwachhöfer and S. Merkulov ([**31**], [**32**], [**33**]) classified irreducible holonomy groups of torsion free connections which are not necessarily compatible with a metric.

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