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MORSE INTERPOLATION FOR HAMILTONIAN GKM SPACES

Catalin Zara

Abstract

Let M be a compact Hamiltonian $T-$space, with finite fixed point set M^T. An equivariant class is determined by its restriction to M^T, and to each fixed point $p \in M^T$ and generic component of the moment map, there corresponds a canonical class τ_p. For a special class of Hamiltonian $T-$spaces, the value $\tau_{p,q}$ of τ_p at a fixed point q can be determined through an iterated interpolation procedure, and we obtained a formula for $\tau_{p,q}$ as a sum over ascending chains from p to q. In general the number of such chains is huge, and the main result of this paper is a procedure to reduce the number of relevant chains, through a systematic degeneration of the interpolation direction. The resulting formula for $\tau_{p,q}$ resembles, via the localization formula, an integral over a space of chains, and we prove that, for complex Grassmannians, $\tau_{p,q}$ can indeed be expressed as the integral of an equivariant form over a smooth Schubert variety.

1. Hamiltonian GKM Spaces

1.1. Equivariant cohomology of Hamiltonian GKM spaces. Let T be a torus and let (M, ω) be a connected, compact, Hamiltonian $T-$space, with finite fixed point set M^T, and moment map $\phi : M \to \mathfrak{t}^*$, where $\mathfrak{t}^*$ is the dual of the Lie algebra of T. Let $H_T^*(M) = H_T^*(M;\mathbb{R})$ be the $T-$equivariant cohomology of M; then $H_T^*(M)$ is a free module over $H_T^*(pt) = \mathbb{S}(\mathfrak{t}^*)$, the symmetric algebra of $\mathfrak{t}^*$. The main purpose of this paper is to give an explicit combinatorial construction of a basis of $H_T^*(M)$ as a module, for a special class of Hamiltonian $T-$spaces.

Hamiltonian $T-$spaces are equivariantly formal. The inclusion map $i : M^T \hookrightarrow M$ induces an injective map $i^* : H_T^*(M) \to H_T^*(M^T)$, and

$$H_T^*(M^T) = \bigoplus_{p \in M^T} H_T^*(p) = \bigoplus_{p \in M^T} \mathbb{S}(\mathfrak{t}^*) = \mathrm{Maps}(M^T, \mathbb{S}(\mathfrak{t}^*)).$$

Hence one can regard a class $f \in H_T^*(M)$ as a map that attaches a polynomial $f_p \in \mathbb{S}(\mathfrak{t}^*)$ to each fixed point $p \in M^T$, and for this reason we will refer to equivariant cohomology classes just by specifying their

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