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GRAFTING, PRUNING, AND THE ANTIPODAL MAP ON MEASURED LAMINATIONS

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Abstract

Grafting a measured lamination on a hyperbolic surface defines a self-map of Teichmüller space, which is a homeomorphism by a result of Scannell and Wolf. In this paper we study the large-scale behavior of pruning, which is the inverse of grafting.

Specifically, for each conformal structure $X \in \mathscr{T}(S)$, pruning X gives a map $\mathscr{ML}(S) \to \mathscr{T}(S)$. We show that this map extends to the Thurston compactification of $\mathscr{T}(S)$, and that its boundary values are the natural *antipodal involution* relative to X on the space of projective measured laminations.

We use this result to study Thurston's grafting coordinates on the space of $\mathbb{CP}^1$ structures on S. For each $X \in \mathscr{T}(S)$, we show that the boundary of the space $P(X)$ of $\mathbb{CP}^1$ structures on X in the compactification of the grafting coordinates is the graph $\Gamma(i_X)$ of the antipodal involution $i_X : \mathbb{P}\mathscr{ML}(S) \to \mathbb{P}\mathscr{ML}(S)$.

1. Introduction

Grafting is a procedure that begins with a hyperbolic structure $Y \in \mathscr{T}(S)$ in the Teichmüller space of a surface S of negative Euler characteristic and a measured geodesic lamination $\lambda \in \mathscr{ML}(S)$. By replacing λ with a thickened version that carries a natural Euclidean metric, a new conformal structure $X = \mathrm{gr}_\lambda Y \in \mathscr{T}(S)$, the *grafting of* Y *along* λ, is obtained.

Scannell and Wolf have shown that for each lamination $\lambda \in \mathscr{ML}(S)$, the conformal grafting map $\mathrm{gr}_\lambda : \mathscr{T}(S) \to \mathscr{T}(S)$ is a homeomorphism, and thus there is an inverse or *pruning map* $\mathrm{pr}_\lambda : \mathscr{T}(S) \to \mathscr{T}(S)$ [**SW**].

In this paper we describe the large-scale behavior of pruning $X \in \mathscr{T}(S)$ in terms of the conformal geometry of X. This description is based on the map

$$\Lambda : Q(X) \to \mathscr{ML}(S)$$

which records the measured lamination equivalent to the horizontal foliation of a holomorphic quadratic differential. Hubbard and Masur

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