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COCYCLE SUPERRIGIDITY AND BOUNDED COHOMOLOGY FOR NEGATIVELY CURVED SPACES

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1. Introduction

In this paper and its companion [31], we introduce new techniques and results in an attempt to extend rigidity theory beyond the scope of linear groups. Amongst our main tools is the bounded cohomology theory recently developed by Burger and Monod [9], [30]. This theory had previously been used for rigidity by Bestvina–Fujiwara, Burger, Iozzi and Monod [2], [7], [8], [9], [22], [30], building on invariants in bounded cohomology with trivial coefficients. Our approach relies in an essential way on bounded cohomology with non-trivial coefficients. It involves, among other things, general cohomology vanishing and non-vanishing results which seem of independent interest. We also make use of a measurable boundary construction introduced by Burger–Monod (loc. cit., later improved by Kaimanovich [25]), which enables us to apply boundary theory for general locally compact groups.

Margulis' fundamental superrigidity theorem may be viewed as a result describing the finite dimensional representation theory of higher rank lattices. Although its original proof was measure theoretic, later remarkable developments of geometric rigidity (cf. [19] and the references therein) were able to approach it as well, at least in the Archimedean co-compact cases. Meanwhile, a rather parallel direction to the geometric one, both in methods and in applications, arose with the appearance of Zimmer's cocycle superrigidity theorem. At the conceptual level, that result, which proved very powerful in various applications, placed the emphasis on the ambient group itself rather than the lattice.

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