

**Erratum to**  
**“New Techniques for Classifying Williams Solenoids”**  
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We regret that we have found a mistake in our paper [1]. The error lay not in our independent theorems, but in our statement of a result in [2]. In the following discussion, we describe the setting for the original paper, the incorrect theorem, and we give an explicit counterexample, which we feel is new and of independent interest. We note that the error did not affect any of our conclusions, and was not actually used in our paper.

A **presentation** is a mapping pair  $(K, f)$  such that  $f : K \rightarrow K$  is a continuous endomorphism of a graph  $K$ , with inverse limit space  $\varprojlim(f, K)$  whose shift map  $\bar{f}$  is conjugate to the given homeomorphism on  $\mathcal{S}$ , a generalized solenoid. Williams showed (in [2, Theorem 3.3]) that two shift maps  $\bar{f}_1$  and  $\bar{f}_2$ , on presentations  $(K_1, f_1)$  and  $(K_2, f_2)$ , are topologically conjugate if and only if the maps  $f_1$  and  $f_2$  are shift equivalent. He was able to show further that shift equivalence is equivalent to “strong shift equivalence” in the category of maps on branched 1-manifolds satisfying the Williams axioms (see [1] or Axioms 2.1 in [2]). This reduces checking shift equivalence to seeking a sequence of “elementary” (or lag 1) shift equivalences.

This erratum treats Williams’ efforts to link the shift equivalence of pointed presentations (corresponding to pointed conjugacy classes of shifts  $\bar{f} : (\varprojlim(K, f), \bar{x}) \rightarrow (\varprojlim(K, f), \bar{x})$ ,  $\bar{x} = (x, x, \dots)$ ) to the shift equivalence of  $\pi_1$  representations. Williams defines the *shift class*  $S(\bar{f}, x)$  of  $\bar{f}$  to be the shift equivalence class of  $\pi_1(f, x) : \pi_1(K, x) \rightarrow \pi_1(K, x)$ . A presentation  $(K, f)$  is **elementary** if  $K$  is a wedge of circles and  $f$  fixes the branch point  $y$  of  $K$ .

Here is the *incorrect* theorem that we had in our paper.

**THEOREM.** *Suppose the elementary presentations  $(K_i, f_i)$ ,  $i = 1, 2$ , satisfy the Williams axioms, and  $f_i(y_i) = y_i$ ,  $i = 1, 2$ . There is a pointed conjugacy*

$$\bar{f} : (\varprojlim(K_1, f_1), \bar{y}_1) \rightarrow (\varprojlim(K_2, f_2), \bar{y}_2)$$