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Invariant Functions on p-divisible Groups and the p-adic Corona Problem

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1. Introduction

In this note we are concerned with p-divisible groups $G = (G_\nu)$ over a complete discrete valuation ring R. We assume that the fraction field K of R has characteristic zero and that the residue field $k = R/\pi R$ is perfect of positive characteristic p.

Let C be the completion of an algebraic closure of K and denote by $\mathfrak{o} = \mathfrak{o}_C$ its ring of integers. The group $G_\nu(\mathfrak{o})$ acts on $G_\nu \otimes \mathfrak{o}$ by translation. Since $G_\nu \otimes K$ is étale the $G_\nu(C)$-invariant functions on $G_\nu \otimes C$ are just the constants. Using the counit it follows that the natural inclusion

$$\mathfrak{o} \xrightarrow{\sim} \Gamma(G_\nu \otimes \mathfrak{o}, \mathcal{O})^{G_\nu(\mathfrak{o})}$$

is an isomorphism. We are interested in an approximate $\bmod \pi^n$-version of this statement. Set $\mathfrak{o}_n = \mathfrak{o}/\pi^n\mathfrak{o}$ for $n \geq 1$. The group $G_\nu(\mathfrak{o})$ acts by translation on $G_\nu \otimes \mathfrak{o}_n$ for all n.

THEOREM 1. *Assume that the dual p-divisible group G' is at most one-dimensional and that the connected-étale exact sequence for G' splits over $\mathfrak{o}$. Then there is an integer $t \geq 1$ such that the cokernel of the natural inclusion*

$$\mathfrak{o}_n \hookrightarrow \Gamma(G_\nu \otimes \mathfrak{o}_n, \mathcal{O})^{G_\nu(\mathfrak{o})}$$

is annihilated by p^t for all ν and n.

The example of $\boldsymbol{G}_m = (\mu_{p^\nu})$ in section 2 may be helpful to get a feeling for the assertion.

In the last section, at the suggestion of the referee we explain the reasoning which led to the statement of the theorem.

We expect theorem 1 to hold without any restriction on the dimension of G as will be explained later. Its assertion is somewhat technical but the proof may be of interest because it combines some of the main results of Tate on p-divisible groups with van der Put's solution of his one-dimensional p-adic Corona problem.

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