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Some Results on Additive Number Theory III

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§1. The main theorem.

Let k be an integer >1 and l be a positive integer. Let $\{P_{ij}: i=1, \cdots, k; j=1, \cdots, l\}$ be a given family of sets, each consisting of prime numbers, subject to the following conditions:

(C_1) For each $i=1, \cdots, k$, the sets $P_{ij}(j=1, \cdots, l)$ are pairwise disjoint;

(C_2) As $x \to \infty$,

$$\sum_{p \leqq x, p \in P_{ij}} \frac{1}{p} = \lambda_{ij} \log\log x + o\,(\sqrt{\log\log x})$$

with positive constants λ_{ij} for $i=1, \cdots, k; j=1, \cdots, l$.

The sets P_{ij} with distinct i's need not be disjoint, and $P_{i1} \cup \cdots \cup P_{il}$ may not contain all primes.

Throughout the paper, without repeated comment, the double subscripts ij will always run through the kl pairs of integers $i=1, \cdots, k$; $j=1, \cdots, l$.

Let $\omega_{ij}(n)$ denote the number of distinct prime factors of a positive integer n, which belong to the set P_{ij}:

$$\omega_{ij}(n) = \sum_{p \mid n, p \in P_{ij}} 1 .$$

THEOREM 1. *Let E be a Jordan-measurable set, bounded or unbounded in the space R^{kl} of kl dimensions. For sufficiently large integer N, let $A(N; E)$ denote the number of representations of N as the sum of k positive integers: $N = n_1 + \cdots + n_k$ such that, if we put*

$$x_{ij} = \frac{\omega_{ij}(n_i) - \lambda_{ij} \log\log N}{\sqrt{\lambda_{ij} \log\log N}} ,$$

the point $(x_{11}, \cdots, x_{1l}, \cdots, x_{k1}, \cdots, x_{kl})$ belongs to the set E. Then, as $N \to \infty$, we have

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