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An Asymptotic Formula for Cayley's Double Partition Function $p(2;n)$

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Introduction

A *double partition* of a positive integer n is a representation of n as a double sum

$$\begin{aligned} n &= a_1+\cdots\cdots+a_k\ ; \\ a_1 &= b_{11}+\cdots\cdots+b_{1m_1}\ , \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ a_i &= b_{i1}+\cdots\cdots+b_{im_i}\ , \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ a_k &= b_{k1}+\cdots\cdots+b_{km_k}\ , \end{aligned}$$

where the *rough parts* $a_1, \cdots, a_k$ and all *fine parts* $b_{i1}, \cdots, b_{im_i}$ are positive integers. The order of each of rough parts and fine parts is disregarded. The numbers $k, m_1, \cdots, m_k$ are arbitrary positive integers. We may express this double partition in the form

$$n=(b_{11}+\cdots+b_{1m_1})+\cdots+(b_{k1}+\cdots+b_{km_k})\ .$$

Thus

$$3=(3)=(2+1)=(1+1+1)=(2)+(1)=(1+1)+(1)=(1)+(1)+(1)$$

has 6 double partitions.

Let us denote the numbers of ordinary and double partitions of n by $p(n)$ and $p(2;n)$ respectively. The functions $p(n)$ and $p(2;n)$ have the relation

$$p(2;n)=\sum_{\substack{s_1,s_2,\cdots\geq 0 \\ 1\cdot s_1+2\cdot s_2+\cdots=n}} \prod_{i=1}^{n} {}_{p(i)}\mathrm{H}_{s_i}\ , \tag{0.1}$$

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