TOKYO J. MATH.
VOL. 3, NO. 2, 1980

On Regular Fréchet-Lie Groups, I
Some Differential Geometrical Expressions of Fourier-Integral Operators on a Riemannian Manifold

Hideki OMORI, Yoshiaki MAEDA and Akira YOSHIOKA

Okayama University, Keio University and Tokyo Metropolitan University

Preface

In this series of papers, we are going to construct a theory of infinite dimensional Lie groups, which will be called "regular Fréchet-Lie groups" throughout this series. Roughly speaking a regular Fréchet-Lie group is a Fréchet-Lie group (i.e., a Lie group modeled on a locally convex Fréchet space) on which product integrals can be defined.

For instance, consider a continuous curve $A(t)$ in the space $M(n)$ of $n \times n$ real matrices, which is naturally regarded as the Lie algebra of the general linear group $\mathrm{GL}(n)$. It is well-known that the solution $Z(t)$ of

$$\frac{d}{dt}Z(t)=A(t)Z(t)\ , \quad Z(0)=I \text{ (the identity)}$$

is given as follows: Let $\Delta=\{t_0, t_1, \cdots, t_n\}$ be a division of the interval $[0, t]$ and let $|\Delta|=\max|t_j-t_{j-1}|$. Then,

$$\lim_{|\Delta|\to 0} \prod_{j=0}^{n-1} \exp\,(t_{j+1}-t_j)A(t_j)$$

converges, and the limit which is written as $\prod_0^t (1+A(s))ds$ gives the solution $Z(t)$. $\prod_0^t (1+A(s))ds$ is called a *product integral* of $A(t)$. Remark that for each fixed t, $h(s, t)=\exp sA(t)$ is a smooth curve in $\mathrm{GL}(n)$ satisfying

$$(*) \qquad \begin{cases} h(s, t)\equiv I\ , \quad \left.\dfrac{\partial}{\partial s}\right|_{s=0} h(*, t)=A(t)\ , \\ h(s, t) \text{ is } C^1 \text{ in } s\ , \\ \dfrac{\partial h}{\partial s}(s, t) \text{ is continuous in } (s, t)\ . \end{cases}$$

Received February 15, 1980