

Analytic Functionals with Non-compact Carrier

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Introduction.

For a compact set L in the complex number plane C , we denote by $\mathcal{O}(L)$ the space of germs of holomorphic functions on L . It is well known that the space $\mathcal{O}(L)$ can be equipped with the topology of DFS space. The dual space $\mathcal{O}'(L)$ of $\mathcal{O}(L)$ is called the space of analytic functionals with carrier in L and was studied extensively by many authors. The aim of this paper is to extend the theory of analytic functionals to the case where L is not compact. For the simplicity, we suppose in this paper L is a closed strip of finite width.

In §1 we introduce the fundamental space $Q(L; K')$ of germs of holomorphic functions. We define in §2 a new series of spaces of holomorphic functions, which will be used to describe the complex representations of the space $Q'(L; K')$ of analytic functionals with carrier in L and of exponential type in K' . §3 treats the Cauchy transformation and we obtain a complex representation of $Q'(L; K')$. In §§4 and 5, we will study the case where L is a closed right strip and that find the situation is very similar to the classical theory of analytic functionals. We will show, among others, the image of the Fourier transformation of $Q'(L; K')$ is the space $\text{Exp}(\mathbf{R} + i(-\infty, -k'_2); L)$ of holomorphic functions of exponential type L defined on the open half plane $\mathbf{R} + i(-\infty, -k'_2)$.

In the final section, we will treat the case where L is an entire strip $L = \mathbf{R} + iK$. Our space of analytic functionals $Q'(L; K')$ is a subspace of the space of Fourier ultra-hyperfunctions (Park-Morimoto [6]), which were first introduced by Sebastião e Silva [8] under the name of ultra-distributions of exponential growth. If $L = \mathbf{R}$ and $K' = \{0\}$, then our space reduces to the space of Fourier hyperfunctions introduced by M. Sato [7] and studied by T. Kawai [2]. Using the results obtained in §§4 and 5, we will study the relation of two definitions of the Fourier transformation of $Q'(L; K')$, one is by the duality and other is by the