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An Analogue of Paley-Wiener Theorem on Rank 1 Semisimple Lie Groups II

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Introduction

This paper is a continuation of the previous paper [8]. Let G be a connected semisimple Lie group with finite center. We assume that G is not compact and moreover, the real rank of G is one. In [8], we have obtained an analogue of Paley-Wiener theorem on $\mathscr{C}(G, \tau)$ (see Theorem 2 in [8]). However, in that theorem we did not consider the discrete part of $\mathscr{C}(G, \tau)$, i.e., the space of τ-spherical cusp forms on G. Therefore in this paper we shall characterize the discrete part of compactly supported functions on G. Here we note that this characterization depends on $\{E_p; 1\leqq p\leqq \gamma\}$ and does not depend on any choice of $\{h_p; 1\leqq p\leqq \gamma\}$ (see (4.12) and (4.15) in [8]). Next using the proof of Theorem 2 in [8], we shall obtain the relation between a size of a support of a compactly supported function on G and an exponential type of its Fourier transform. To obtain the relation we shall use the same method in the classical Paley-Wiener theorem on an Euclidean space.

In §2 using the results of Harish-Chandra [6], we shall reform the theorem of J. Arthur [1] and obtain some characterization of $\mathscr{C}(G)$. Then applying the above consideration to each K-finite subspace of $\mathscr{C}(G)$, we shall obtain an analogue of Paley-Wiener theorem on $\mathscr{C}(G)$ (see §3).

§1. More precise characterization.

For an arbitrary function g in $\mathscr{C}(G, \tau)$ we shall define g' by $\mathscr{C}_A^{-1}(\mathscr{C}_A(g))$ and g° by $g-g'$. Then from Theorem 1 in [8] we can easily prove that g' belongs to $\mathscr{C}_A(G, \tau)$ and g° to ${}^\circ\mathscr{C}(G, \tau)$. Let notation be as in [8].

LEMMA 1. (i) ${}^\circ\mathscr{C}(G, \tau)$ *is contained in the space which is generated*

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