TOKYO J. MATH.
VOL. 3, NO. 1, 1980

On the Genus Fields of Pure Number Fields

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In this paper, we shall investigate the genus fields of pure number fields $K=\boldsymbol{Q}(\sqrt[n]{a})$ with $a\in\boldsymbol{Z}$. In the case of odd n, the genus fields of such number fields K are explicitly determined in Fröhlich [1] and Ishida [3]. On the other hand, in the case of even n, the situations are somewhat complicated, as is expected from the genus theory of quadratic number fields. Here we shall show some partial results for the even degree cases and also give a new elementary proof for the odd degree cases. As for the definitions and some fundamental properties of genus fields, see Ishida [3].

§ 1. Preliminaries.

Let $K=\boldsymbol{Q}(\sqrt[n]{a})$ with $a\in\boldsymbol{Z}$ $(a\neq\pm1)$ be a pure number field, where a has the property

$$(*)\qquad p^{v}\,\|\,a\Longrightarrow(v,\,n)=1$$

for any prime divisor p of a. Then, of course, the degree of K over $\boldsymbol{Q}$ is n. Let

$$n=q_0^{s_0}q_1^{s_1}q_2^{s_2}\cdots q_t^{s_t}\quad(s_0\geqq0;\,s_1,\,s_2,\,\cdots,\,s_t>0)\ ,$$

where $q_0=2$ and q_i $(i=1,\,2,\,\cdots,\,t)$ are odd primes, and put

$$K_i=\boldsymbol{Q}((\sqrt[n]{a})^{n/q_i^{s_i}})\qquad(i=0,\,1,\,\cdots,\,t)\ .$$

Then we have

$$K=K_0\cdot K_1\cdot K_2\cdot\ \cdots\ \cdot K_t\quad(\text{composite})\ .$$

Now let K^* be the genus field of K and k^* the maximal abelian subfield of K^*: $K^*=k^*K$. Also let $k^{(i)*}$ $(i=0,\,1,\,\cdots,\,t)$ be the maximal abelian subfield of the genus field of K_i. Since the degrees of K_i over $\boldsymbol{Q}$ are coprime to each other, we have

Received September 4, 1979