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The Cauchy Problem for Weakly Hyperbolic Equations (II); Infinite Degenerate Case

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Introduction

In this paper we shall deal with well-posedness of the Cauchy problem for some weakly hyperbolic operators with involutive and non-involutive multiple characteristics. For the second order equations, Oleinik [4] obtained a sufficient condition for well-posedness. Menikoff [1] extended Oleinik's results to the equations of higher order, and Ohya [3] improved and simplified Menifoff's proof. In a previous paper [5], we considered weakly hyperbolic operators whose characteristic roots come across one another with finite order at $t=0$.

Recently Nishitani [2] has proved well-posedness of the Cauchy problem for a hyperbolic operator with characteristic roots coming into double at $t=0$ in contact with each other of infinite order. In this article we shall treat the case where the characteristic roots may have $m(\geqq 2)$ multiplicities at $t=0$ in contact with one another of infinite order.

Now our concern is the following operator P which is a partial differential operator of the form:

$$P=D_t^m+\sum_{\substack{|\alpha|+j\leqq m\\ j\leqq m-1}} a_{\alpha,j}(x,t)D_x^\alpha D_t^j$$

where each coefficient $a_{\alpha,j}(x,t)$ belongs to $\mathscr{B}((0,T)\times \boldsymbol{R}^n)$. Let $\lambda_j(x,t,\xi)$, $j=1,\cdots,m$, be the characteristic roots of P. If all the $\lambda_j(x,t,\xi)$ are real valued functions in $\mathscr{B}((0,T), S^1)$ for some $T>0$, P is said to be a weakly hyperbolic operator. We consider only such operators throughout the paper.

We say that a weakly hyperbolic operator P has involutive characteristic roots if for any i, $j(1\leqq i, j\leqq m)$, there exist pseudo-differential operators $A_{i,j}$, $B_{i,j}$ and $C_{i,j}\in\mathscr{B}((0,T), S^0)$ such that

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