TOKYO J. MATH.
VOL. 4, NO. 2, 1981

On Normal Projective Surfaces with Trivial Dualizing Sheaf

Yumiko UMEZU

Tokyo Metropolitan University

(Communicated by K. Ogiue)

Let X be a normal projective surface defined over an algebraically closed field k of characteristic $\neq 2, 3$, whose dualizing sheaf ω_X is isomorphic to the structure sheaf $\mathcal{O}_X$. In this paper we shall investigate such a surface, especially when it has at least one singular point with positive geometric genus (see Definition 1).

Normal surface singularities with small geometric genera have been studied by many authors (cf. Artin [2], Laufer [7] and Yau [11]). Their results show that such singularities have rather simple properties, and it seems that the geometric genus is an important invariant for studies of singularities.

We will determine the geometric genus of singular points on X by means of the irregularity q of a non-singular model of X (Theorem 1), and estimate q in terms of the dimension of the projective space in which X is embedded (Theorem 3). On the other hand, as a corollary to Theorem 1, we have $H^1(X, \mathcal{O}_X)=0$ if X is not an abelian surface, and hence such an X has properties similar to a $K3$ surface; a characterization of X (when X is not an abelian surface) is given in Theorem 2.

The author would like to express her thanks to K. Watanabe and F. Hidaka for their useful suggestions and encouragements and to Y. Miyaoka whose suggestion about a "double section" of an elliptic ruled surface was a great help to prove Lemma 2.

§ 1. Preliminaries.

In this section we summarize some local properties of singularities on a normal surface. Let X be a normal surface with a singular point x. Let $\pi: \tilde{X} \to X$ be the minimal resolution of x and let $A = \bigcup_{i=1}^{n} A_i$ denote the exceptional set $\pi^{-1}(x)$, where A_i's are the irreducible components of A.

Received October 13, 1980