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On Regular Fréchet-Lie Groups IV

Definition and Fundamental Theorems

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Introduction

In the previous papers [10], [11], [12], we have seen that Fourier integral operators on a compact manifold have some group theoretical characters. Indeed, one of the purposes of this series is to show that the group of all invertible Fourier integral operators of order 0 on a C^∞ compact riemannian manifold is an *infinite dimensional Lie group.* It should, however, be remarked that we have not given in the previous papers the definition of infinite dimensional Lie groups. It will be given in this paper, hence *one may read this paper without knowing anything about the previous papers.*

Now, it continues to be a basic question when one may call a group G an infinite dimensional Lie group. However, taking the basic properties of finite dimensional Lie groups in mind, we suggest the following (L1)$\sim$(L3) are necessary at least, where

(L1) G is a C^∞ infinite dimensional manifold and the tangent space $\mathfrak{g}$ at the identity has a Lie algebra structure, called the Lie algebra of G.

(L2) There exists the exponential mapping exp of $\mathfrak{g}$ into G such that $\{\exp tu;\ t\in \mathbf{R}\}$ is a smooth one parameter subgroup of G for every $u\in\mathfrak{g}$.

(L3) Local group structures of G (i.e., a neighborhood of the identity) can be determined by its Lie algebra $\mathfrak{g}$.

Hilbert or Banach-Lie groups [1], [5] satisfy these conditions and so do strong *ILB*- (or strong *ILH*-) Lie groups defined by Omori [8], [9].

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