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The Centralizers of Semisimple Elements of the Chevalley Groups E_7 and E_8

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The purpose of this paper is to give in detailed tables all the centralizers and their orders of semisimple elements of the finite Chevalley groups E_7 and E_8. These tables are very useful since they give also the character degrees of the semisimple irreducible complex representations constructed by Deligne and Lusztig [7] for the finite Chevalley groups of adjoint type. For, these degrees can be obtained if we know what subgroups of the finite Chevalley groups of universal type are centralizers of semisimple elements (see [7]). Similar tables giving these centralizers for the classical groups and for the groups G_2, F_4 and E_6 have been obtained in [5], [6], [12] and [11] respectively.

A considerable amount of detailed work was involved in the compilation of our tables which has not been included in the paper. However we outline below the general results on which we relied heavily for our calculations.

Let G be a simple linear algebraic Chevalley group of rank l defined over the algebraic closure K of the prime field F_p of p elements. Let Φ be a root system of G with respect to a maximal torus T_0 of G which splits over F_p. Consider the highest root r_0 in Φ and let $\tilde{\Delta}=\Delta\cup\{-r_0\}$ where $\Delta=\{r_i;\ i=1, \cdots, l\}$ is a fixed fundamental basis of Φ. Also we put $I_0=\{0, 1, 2, \cdots, l\}$.

We have shown [8] that, except for the bad primes of G (see [1, p. 178]), a connected reductive subgroup G_1 of maximal rank in G is the connected centralizer of a semisimple element if and only if some proper subset of the roots in $\tilde{\Delta}$ is equivalent under the Weyl group $W(=W(\Phi))$ to a fundamental basis of the root system of G_1. Thus every connected centralizer of a semisimple element in G is in some C_J, $J\subsetneqq I_0$, where by C_J we denote the set of all connected centralizers of semisimple elements

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