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On the Sequential Approximation of Scalarly Measurable Functions by Simple Functions

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This paper is concerned with the approximation of weak*-measurable functions by means of simple functions. Let X be a Banach space, X^* the dual space of X, and let (S, Σ, μ) be a finite nonnegative complete measure space. In this paper we mainly treat with X^*-valued functions defined on the set S; hence we call (S, Σ, μ) the base measure space in the following. A function $f: S \to X^*$ is said to be weak*-measurable if for every $x \in X$ the numerical function $\langle x, f \rangle$ is μ-measurable. This kind of definition of measurability for X^*-valued functions does not assume the existence of approximate sequence of simple functions and is generically called a scalar measurability. On the other hand, a function $f: S \to X^*$ is said to be strongly measurable if there exists a sequence (f_n) of simple functions with $\lim \|f_n(s) - f(s)\| = 0$ a.e.; hence the existence of an approximate sequence (f_n) of simple functions is involved in the definition itself. By use of the martingale argument, it is possible to find for every norm-bounded weak*-measurable function $f: S \to X^*$ a generalized sequence (f_α) of simple functions approximating f in the sense that $\lim_\alpha \| \langle x, f_\alpha \rangle - \langle x, f \rangle \|_{L^1(\mu)} = 0$ for each $x \in X$. However, the weak*-measurability of a function f does not necessarily imply the existence of a sequence (f_n) of a countable number of simple functions that approximate f in the sense that

$$(*) \qquad \langle x, f_n \rangle \longrightarrow \langle x, f \rangle \quad \text{a.e.}$$

for each $x \in X$, where the null set on which the convergence does not hold may vary with x. More precisely, such sequential approximation of weak*-measurable functions by simple functions need not be possible unless the Banach space X or else the base measure space (S, Σ, μ) is suitably chosen. We will see later that even a weakly measurable function does not necessarily have approximate sequences if the base

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