TOKYO J. MATH.
VOL. 6, NO. 1, 1983

Configurations and Invariant Gauss-Manin Connections for Integrals II

Kazuhiko AOMOTO

Nagoya University
(Communicated by T. Saito)

In this note, by using certain canonical invariant 1-forms, we shall compute explicitly the formulae of Gauss-Manin connections for the integrals which have been investigated in [1] and prove [3] Theorem 2 in the hyperquadric case (see Theorems 1, 2 and 3). We shall follow the terminologies used in [1].

§1. Conformal case.

First, we are going to find out an explicit representation of invariant Gauss-Manin connection for the following integral which admits conformal transformations:

$$(1.1)\qquad \hat{\varphi}(\phi)(=\hat{\varphi}(\lambda_0, \lambda_1, \cdots, \lambda_m; \phi)) \\ =\int (x_1^2+\cdots+x_n^2+1)^{\lambda_0}\hat{f}_1(x)^{\lambda_1}\cdots\hat{f}_m^{\lambda_m}(x)dx_1\wedge dx_2\wedge\cdots\wedge dx_n\ ,$$

where $\hat{f}_1, \cdots, \hat{f}_m$ denote general linear functions: $\hat{f}_j=\sum_{\nu=1}^n u_{j\nu}x_\nu + u_{j0}$. Here we use the variables $\sqrt{-1}\,x_\nu$ instead of x_ν in [1] (J, III_0).

As in [1], we define the symmetric matrix of $(m+1)$ order $A=((a_{ij}))$ $0\leqq i, j\leqq m$ by putting $a_{ij}=\sum_{\nu=0}^n u_{i\nu}u_{j\nu}$ and $a_{i0}=a_{0i}=u_{i0}$, where we normalize $a_{ii}=1$. We denote by $A\binom{I}{J}$ the minor determinant of the $i_1, \cdots, i_p$-th rows and the $j_1, \cdots, j_p$-th columns for $I=(i_1, \cdots, i_p)$ and $J=(j_1, \cdots, j_p)$, and write $A(I)$ for $A\binom{I}{I}$.

It has been stated in [1] Lemma 3.3 that the above integrals have a basis of functions:

$$(1.2)\qquad \hat{\varphi}(i_1\cdots i_p)=\hat{T}_{i_1}^{-1}\cdots\hat{T}_{i_p}^{-1}\hat{\varphi}(\phi)=\int \hat{U}(\lambda)\frac{dx_1\wedge\cdots\wedge dx_n}{\hat{f}_{i_1}\cdots\hat{f}_{i_p}}$$

Received April 26, 1982
Revised October 12, 1982