TOKYO J. MATH.
VOL. 6, No. 2, 1983

Periodic Solution of Classical Hamiltonian Systems

Kiyoshi HAYASHI

Keio University

Introduction

Let $x=(x_1, x_2, \cdots, x_n)$, $y=(y_1, y_2, \cdots, y_n)\in \boldsymbol{R}^n$ and $H=H(x, y)$: $\boldsymbol{R}^{2n}\to\boldsymbol{R}$ be a smooth (C^∞) function.

We consider a Hamiltonian system of n degrees of freedom

$$(1)\qquad \dot{x}_i=H_{y_i}\ , \quad \dot{y}_i=-H_{x_i}\ ; \quad i=1, 2, \cdots, n\ .$$

P. Rabinowitz [3] proved that "if an energy surface

$$H^{-1}(e)=\{(x, y)\in \boldsymbol{R}^{2n};\ H(x, y)=e\}$$

is star-shaped, then there exists at least one periodic solution of (1) on it".

"star-shaped" implies "diffeomorphic to the sphere S^{2n-1}", but it is not known whether the condition "star-shaped" can be replaced by "diffeo. to S^{2n-1}" or not. This is a generalized Hamiltonian version of the Seifert Conjecture (Has any sufficiently smooth flow on S^3 periodic orbit ?) in the theory of dynamical systems.

Classically, the system (1) is derived from the Lagrangian system and, in the time-independent case, the Hamiltonian H is the sum of the kinetic energy and the potential. So we define

DEFINITION. A Hamiltonian $H=H(x, y)$ is called *classical* if it has the form

$$(2)\qquad H=\sum_{i,j=1}^{n} a^{ij}(x)y_iy_j+U(x)\ ,$$

where a^{ij}, U: $\boldsymbol{R}^n\to\boldsymbol{R}$ are smooth functions and for any $x\in\boldsymbol{R}^n$, the matrix $(a^{ij}(x))$ is symmetric and positive definite.

The system (1) with classical Hamiltonian H is called a *classical Hamiltonian system.*

In this paper we have

Received December 1, 1982