TOKYO J. MATH.
VOL. 6, No. 2, 1983

The Existence of an Invariant Stable Foliation and the Problem of Reducing to a One-Dimensional Map

Michiko YURI

Tsuda College

(Communicated by M. Mori)

Introduction

In 1962, E. Lorenz found the first example of a strange attractor by investigating a hydrodynamical system. Recently, another equation has been proposed by Rössler, and by numerical solution, it was shown that these equations indicate the existence of a two-dimensional attractor which has a compact "ribbon-like" structure.

As the attractor can be treated as a "single-sheeted" quasi-two-dimensional object, we take a cut across the attractor and construct a Poincaré mapping by means of which we can reduce a three-dimensional continuous flow to a one-dimensional discrete mapping. Thus one-dimensional models serve as the simplest example of models for some dynamical systems and have become common. They appear in the original paper by Lorenz [1], and also in more recent works of Guckenheimer [2], Rössler [3], and others ([4], [5], [6]). But this procedure has not been justified rigorously so far. Our purpose here is to give some justification for reducing a three-dimensional flow which has a two-dimensional attractor to a one-dimensional mapping. To be more precise with the problem, let us consider a map H_0: $\mathrm{R}^2 \to \mathrm{R}^2$ defined by

$$H_0(x, y) = (f(x), \mu y)$$

where f is a map of piecewise C^2-class such that $f(I) \subset I$ for an interval I and $0 < \mu < 1$. The map H_0 has trivial stable foliation $\{x = Constant\}$, and hence the behavior of H_0 near the invariant set $I \times \{0\}$ is reduced to the one-dimensional map f on I. Let H be a perturbation of H_0 defined by

Received February 5, 1982
Revised June 23, 1983