On Regular Fréchet-Lie Groups VI

Infinite Dimensional Lie Groups Which Appear in General Relativity

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It is well-known that Newtonian mechanics is an approximation for general relativity. However, mathematically, both mechanics can stand independently without any subordination by each other. That means, without experimental fact one cannot prove mathematically that general relativistic mechanics is more adequate than Newtonian mechanics.

Nevertheless, beside physics, we insist in this paper that under the group theoretical point of view, general relativistic mechanics is a more closed system than Newtonian mechanics.

Introduction

Hamiltonian mechanics is a beautifully organized mathematical expression of classical mechanics (cf. [1]) and is expressed as the triplet (M, Ω, H) , where

- (i) (M, Ω) is a symplectic manifold, called a *phase space*. M is even dimensional smooth manifold, and Ω a smooth symplectic structure on it.
- (ii) H is a smooth function on M, called a Hamiltonian. Mechanical motions governed by the above Hamiltonian H are given by the integral curve of the Hamiltonian vector field X_H , where X_H is defined by

$$\Omega \sqcup X_H = dH$$
.

Let $\varphi_t(x)$ be the integral curve of X_H with an initial point $x \in M$, namely $x_t = \varphi_t(x)$ is the unique solution of the following equation: