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The Grothendieck Group of a Finite Group Which is a Split Extension by a Nilpotent Group

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Introduction

Let R be a ring. Then the Grothendieck group $G_0(R)$ is the abelian group given by generators $[M]$ where M is a finitely generated R-module, with relations $[M]=[M']+[M'']$ whenever $0\to M'\to M\to M''\to 0$ is an exact sequence of finitely generated R-modules. Let π be a finite group, and $\mathcal{O}$ be a maximal order in $\boldsymbol{Q}\pi$ containing $\boldsymbol{Z}\pi$. Then Swan [4] showed that there is a natural epimorphism from $G_0(\mathcal{O})$ onto $G_0(\boldsymbol{Z}\pi)$. He also gave an example of cyclic group such that $G_0(\boldsymbol{Z}\pi)\not\cong G_0(\mathcal{O})$. In connection with these results of Swan, it is an interesting problem to investigate the relation between $G_0(\boldsymbol{Z}\pi)$ and $G_0(\mathcal{O})$. For an abelian group π, Lenstra [1] gives the description of $G_0(\boldsymbol{Z}\pi)$ which answers the above question. Recently, Miyamoto [2] generalizes Lenstra's result into nilpotent groups.

In this paper, we treat a finite group with a normal nilpotent subgroup which has a complement. For such a group π, we obtain an analogous decomposition of $G_0(\boldsymbol{Z}\pi)$.

THEOREM. *Let π be a finite group with a normal nilpotent subgroup U which has a complement. Then we have*

$$G_0(\boldsymbol{Z}\pi)\cong\bigoplus_{e\in Y} G_0\left(\boldsymbol{Z}\pi e^*\left[\frac{1}{d(e)}\right]\right),$$

where Y is a set of the representatives of the π-conjugacy classes of centrally primitive idempotents of $\boldsymbol{Q}U$, e^* denotes the class sum of the class containing e and $d(e)=|U|/|\mathrm{Ker}\,(U\to\boldsymbol{Q}Ue)|$.

REMARK 1. The idempotent e of the ring R is called centrally primitive, if e is a primitive idempotent of the center of the ring R.

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