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Boundary Regularity for Minima of Certain Variational Integrals

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Introduction

Let Ω be a bounded open domain of $\boldsymbol{R}^n$, $n \geqq 2$, with boundary $\partial\Omega$ of class C^2, Γ_1 a relatively open subset, of $\partial\Omega$, and $\Gamma_0 = \partial\Omega - \Gamma_1$. We consider the variational integral

$$F(u) := \int_\Omega f(x, u, Du)dx ,$$

for a function $u: \Omega \to \boldsymbol{R}^N$, where $Du = (\partial u^i/\partial x^\alpha)_{1 \leq i \leq N, 1 \leq \alpha \leq n}$, and $f(x, u, \xi)$: $\Omega \times \boldsymbol{R}^N \times \boldsymbol{R}^{nN} \to \boldsymbol{R}$ is a Carathéodory function; i.e. measurable in x for each $(u, \xi) \in \boldsymbol{R}^N \times \boldsymbol{R}^{nN}$, and continuous in (u, ξ) for almost every $x \in \Omega$.

In this paper we consider the following variational problem with mixed boundary condition:

$$(*) \quad \begin{cases} \text{Find a minimizing function } u: \Omega \to \boldsymbol{R}^N \text{ of } F(u) \text{ which maps } \Gamma_1 \\ \text{into some hyperplane } \Sigma := \{v \in \boldsymbol{R}^N : v^{s+1} = \cdots = v^N = 0\} \text{ and has} \\ \text{prescribed Dirichlet data } \phi \text{ on } \Gamma_0, \text{ where } \phi(\Gamma_0 \cap \bar{\Gamma}_1) \subset \Sigma. \end{cases}$$

(See [1] for the mixed boundary problem for harmonic maps.)

In the paper [4], M. Giaquinta and E. Giusti prove interior regularity of minima of variational integrals (see also [5]). On boundary regularity for Dirichlet problem a result due to J. Jost and M. Meier [8] is known.

In this paper we investigate the behavior of the solution of (*) near Γ_1.

§1. L^p-estimate for the gradient.

We suppose that the function f satisfies the growth condition:

$$(1.1) \qquad a|\xi|^m - k \leqq f(x, u, \xi) \leqq b|\xi|^m + k ,$$

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