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Expansion of the Solutions of a Gauss-Manin System at a Point of Infinity

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Introduction

Let $f(x)$ be a polynomial, in n complex variables $x=(x_1, \cdots, x_n)$, with an isolated critical point and let $F_0(t, x)$ be a deformation of $f(x)$ with parameters $t=(t_1, \cdots, t_m)$. Setting $F=t_0+F_0$ with a distinguished parameter t_0, we shall investigate the differential system to be satisfied by the integral of type

$$(1) \qquad u=\int \delta^{(\lambda)}(F)dx \quad \text{or} \quad \int F^{-\lambda-1}dx \quad (dx=dx_1\wedge\cdots\wedge dx_n)\,,$$

where λ is a (generic) complex number. Roughly speaking, such a *Gauss-Manin system* defines a meromorphic connection, on the space S of parameters (t_0, t), at most with poles along its *discriminant variety* D. Thus, our attension will be paid to the many-valued holomorphic solutions on $S\backslash D$ of the Gauss-Manin system. In "simple" examples, one can show that a fundamental system $\Phi(t_0, t)$ of its many-valued holomorphic solutions can be expanded into a power series

$$(2) \qquad \Phi(t_0, t)=\sum_{r=0}^{\infty}\Phi_r(t)t_0^{-\Lambda-(r+1)I}$$

convergent near the point $(t_0, t)=(\infty, 0)$ at infinity, where $-\Lambda$ is the matrix of exponents of f shifted by λ. In the present article, we shall determine such an expansion of Φ in an explicit manner for typical examples of Gauss-Manin systems.

Our computational results will be given in §3. The polynomial $f(x)$ to be deformed is assumed there to belong to either of the types

$$(3) \qquad \begin{aligned} &(\mathrm{I}) \quad f(x)=x_1^{p_1}+x_2^{p_2}+\cdots+x_n^{p_n} \quad \text{and} \\ &(\mathrm{II}) \quad f(x)=x_1^{p_1}+x_1x_2^{p_2}+x_3^{p_3}+\cdots+x_n^{p_n}\,. \end{aligned}$$

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