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Some Unramified Cyclic Cubic Extensions of Pure Cubic Fields

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Introduction

In [6], Ishida has explicitly constructed the genus field of an algebraic number field F of a certain type. Therefore it is of some interest to construct unramified abelian extensions, of F, which are not contained in the genus field. In this paper, we shall consider this problem in the case that F is a pure cubic field.

Let $\boldsymbol{Q}$ denote the field of rational numbers, and let $\boldsymbol{Z}$ be the ring of rational integers. Let $K=\boldsymbol{Q}(\sqrt[3]{m})$ be a real pure cubic field, where m is a positive cubefree rational integer. Let $\zeta=\exp(2\pi i/3)$. Let $k=\boldsymbol{Q}(\zeta)$ and $\tilde{K}=Kk$. Then $\tilde{K}$ is the Galois closure of K. Let M (resp. M') be the genus field of K (resp. $\tilde{K}$) over $\boldsymbol{Q}$ (resp. k). The field M was given explicitly in [1]. We shall give some unramified cyclic cubic extensions, of K, which are not contained in M. Let $\mathrm{Re}\,\alpha$ denote the real part of a complex number α. Then such extensions are written in the form $K(\mathrm{Re}\sqrt[3]{\varepsilon_0})$, where ε_0 is a unit of $\tilde{K}$ with some properties (cf. Theorems 1.3 and 3.1).

Notations: Let J be the complex conjugate map, and let σ be a generator of $\mathrm{Gal}(\tilde{K}/k)$ with $(\sqrt[3]{m})^{\sigma}=\sqrt[3]{m}\cdot\zeta$, Then $\mathrm{Gal}(\tilde{K}/\boldsymbol{Q})$ is generated by $\{J, \sigma\}$ with the relations $J^2=\sigma^3=1$, $\sigma J=J\sigma^2$. For an algebraic number field F, let F^* (resp. E_F) denote its multiplicative group (resp. its unit group).

§1. Preliminaries.

LEMMA 1.1. *Let $\mathscr{A}$ be the set of all the unramified cyclic cubic extensions of K and let $\mathscr{B}$ be the set of all the unramified cyclic cubic extensions, of $\tilde{K}$, which are abelian over K. (We note from Kummer theory that any element of $\mathscr{B}$ is written in the form $\tilde{K}(\sqrt[3]{\alpha})$, where*

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